TWO TREATMENT ORTHOGONAL 3 × 12 ROW-COLUMN DESIGNS FOR SIX TREATMENTS

BU-799-M W. T. Federer December 1982

Abstract

Two three-row by six-column designs with six treatments which have the two sets of treatments in an orthogonal arrangement are given. This is of interest because this is impossible for two latin squares of order six, even though the same number, 36, of experimental units are involved.

- - - - - -

It is well-known that a pair of orthogonal latin squares of order six does not exist. It was shown in BU-795-M that a pair of orthogonal F-rectangles with six rows and twelve columns for six treatments could be constructed. We demonstrate via construction that one can construct two 3 × 12 row-column designs for six treatments which have the two sets of treatments in an orthogonal arrangement. The treatments and columns, of course, are not in an orthogonal arrangement, since there are only three rows and six treatments.

The two designs with their treatment column incidence matrices are:

Design I:

	Column (Treatments 1, 2, 3, 4, 5, 6)											
Row	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	2	2	3	3	4	4	5	5	6	6
2	2	2	3	3	4	4	5	5	6	6	1	1
3	4	4	5	5	6	6	1	1	2	2	3	3

$$N_{TC} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$N_{TC}N'_{TC} = \begin{bmatrix} 6 & 2 & 2 & 4 & 2 & 2 \\ & 6 & 2 & 2 & 4 & 2 \\ & & 6 & 2 & 2 & 4 \\ & & & 6 & 2 & 2 \\ & & & & 6 & 2 \\ & & & & & 6 \end{bmatrix}$$

Design II:

	Column (Treatments a, b, c, d, e, f)											
Row	1	2	3	4	5	6	7	8	9	10	11	12
1	a	d	b	e	c	f	d	a	e	b	f	c
2	c	f	d	a	e	b	f	c	a	d	b	e
3	f	c	a	d	b	e	c	f	d	a	e	b

$$N_{TC} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$N_{TC}N'_{TC} = \begin{bmatrix} 6 & 2 & 2 & 4 & 2 & 2 \\ & 6 & 2 & 2 & 4 & 2 \\ & & 6 & 2 & 2 & 4 \\ & & & 6 & 2 & 2 \\ & & & & 6 & 2 \\ & & & & & 6 \end{bmatrix}$$

Observing the matrix $N_{TC}N'_{TC}$, we note that the treatments and columns in both designs are in a group divisible partially balanced incomplete block design arrangement. This appears to be the best that one can do toward achieving balance of the columns and treatments. Instead of the numbers 2 and 4 in $N_{TC}N'_{TC}$, one would like to have two 3s and three 2s as this would be closer to balance. It is not known if such designs exist.

The occurrence of treatments in the two row-column designs is:

	Design I					
Design II	1	2	3	4	5	6
a	1	1	1	1	1	1
b	1	1	1	1	1	1
c	1	1	1	1	1	1
d	1	1	1	1	1	1
e	1	1	1	1	1	1
f	1	1	1	1	1	1

It should be noted that Designs I and II each have 36 experimental units, the same number as in a latin square design of order 6. The treatments are in an orthogonal arrangement to each other, which is impossible for latin squares of order six. One has given up column-treatment orthogonality, but has obtained treatment-treatment orthogonality. Row-treatment orthogonality is maintained in that each treatment occurs twice in each row for both designs.

It is not known if more than two treatment-orthogonal 3-row by 12-column designs exist.

Reference

Federer, W. T., A. Hedayat and J. P. Mandeli. (1982). Pairwise orthogonal F-rectangle designs. BU-795-M in the Mimeo Series of the Biometrics Unit, Cornell University.